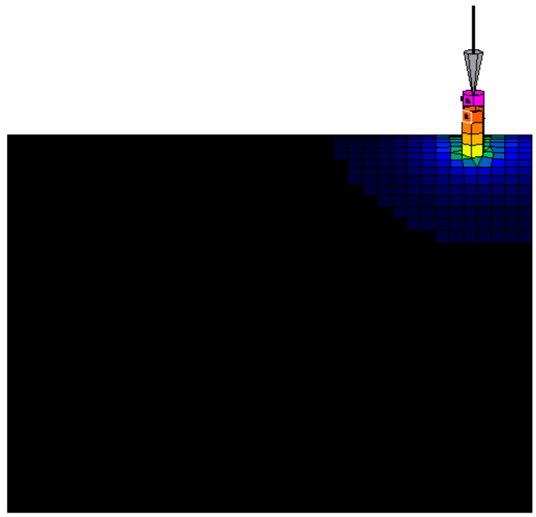
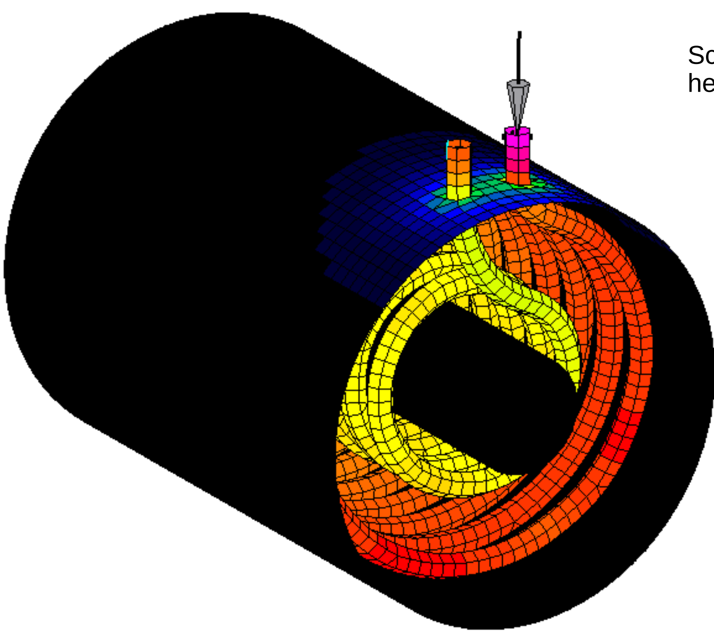
Coil-In-Tube Cross Flow Heat Exchanger

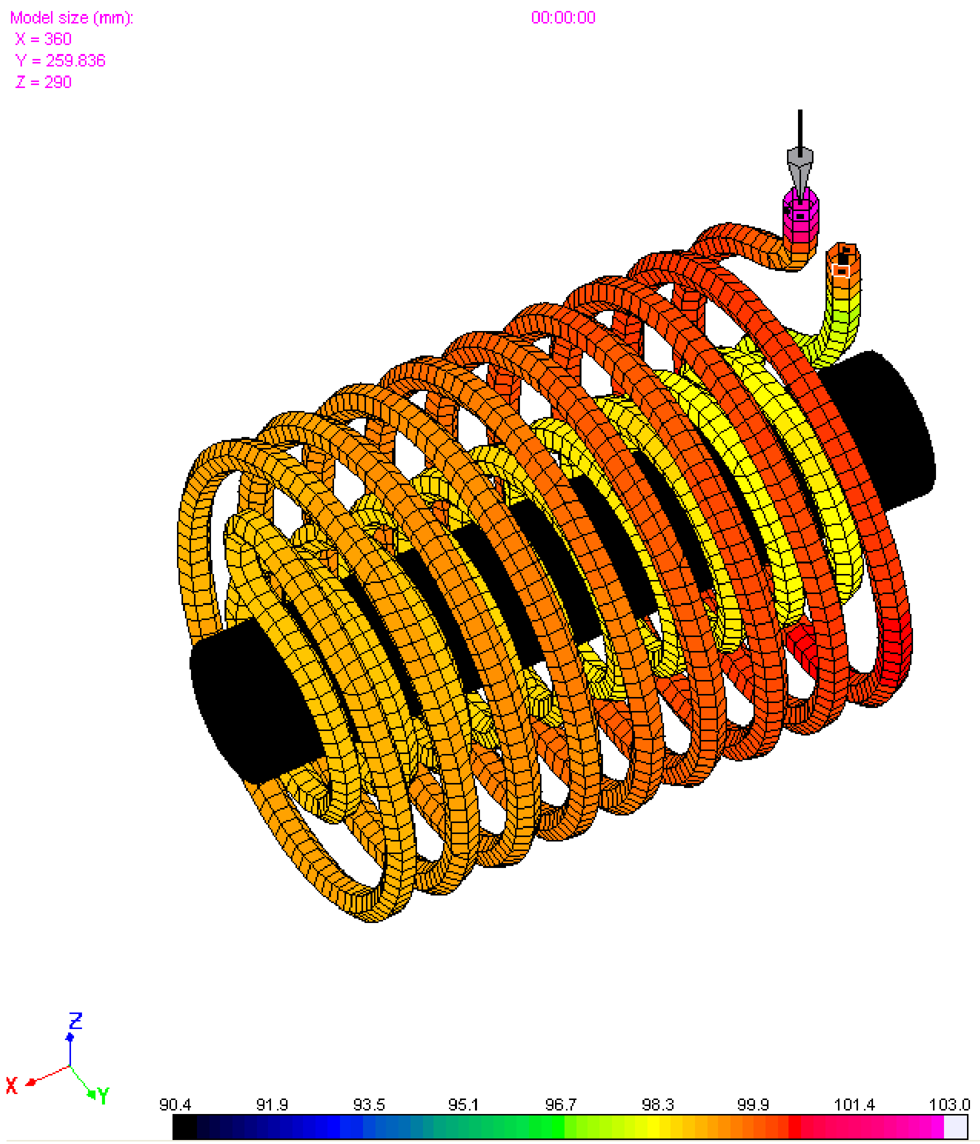
In order to demonstrate the fluid stream and convection heat transfer modeling capabilities of RadTherm, a coil-in-tube cross flow heat exchanger was simulated (shown below).
The heat exchanger has hot engine oil (126 degrees C at the entrance) flowing in the inner coil and water (90 degrees C at the entrance) flowing in the opposite direction in the larger cylinder.

Using the same dimensions and the inlet and outlet temperatures from the simulation, the same heat transfer problem is solved analytically below. In order to keep the analytical solution simple, convection to the outside environment and all radiation was turned off (zero emmisivity). The equations below solve for the overall length of the heat exchanger. The calculated length is compared to the actual (modeled) length.





Screen shots of the whole coil-in-tube heat exchanger taken from RadTherm



Screen shot of the inner coil taken from RadTherm

Inlet and OutletTemperatures (from Radtherm):

$T_{oil.i} := 126 \cdot C$ $\quad$ $T_{oil.o} := 115.8 \cdot C$

$T_{water.i} := 90 \cdot C$ $\quad$ $T_{water.o} := 90.72 \cdot C$

Dimensions of the model heat exchanger:

$D_{helix} := 16 \cdot mm$ $\quad$ $L_{actual} := 10.2 \cdot m$

$D_{cyl.o} := 220 \cdot mm$

$D_{cyl.i} := 120 \cdot mm$

Fluid Properties of the water and oil (taken at mean temperatures):

$$T_{mean.oil} := 390\cdot K \qquad T_{mean.water} := 365\cdot K$$

$$C_{p.oil} := 2294\cdot\frac{J}{kg\cdot K} \qquad \mu_{oil} := .011\cdot N\cdot\frac{s}{m^2} \qquad k_{oil} := .135\cdot\frac{W}{m\cdot K} \qquad Pr_{oil} := 187 \qquad \rho_{oil} := 830.6\cdot\frac{kg}{m^3}$$

$$\mu_{s.oil} := .0186\cdot N\cdot\frac{s}{m^2}$$

$$C_{p.water} := 4209\cdot\frac{J}{kg\cdot K} \quad \mu_{water} := 306\cdot 10^{-6}\cdot N\cdot\frac{s}{m^2} \quad k_{water} := .677\cdot\frac{W}{m\cdot K} \quad Pr_{water} := 1.91 \quad \rho_{water} := 963.4\cdot\frac{kg}{m^3}$$

Calculation of mass flow rates in the inner coil (oil) and outer cylinder (water):

$$V_{oil} := 1.5\frac{L}{min} \qquad m_{oil} := V_{oil}\cdot\frac{m^3}{1000\cdot L}\cdot\rho_{oil}\cdot\frac{1\cdot min}{60s} \qquad m_{oil} = 0.021\frac{kg}{s}$$

$$V_{water} := 10\frac{L}{min} \qquad m_{water} := V_{water}\cdot\rho_{water}\cdot\frac{1\cdot min}{60s} \qquad m_{water} = 0.161\frac{kg}{s}$$

Calculation of Reynolds number, Nusselt number, and convection coefficient for the oil fluid stream:

$$Re_{D.oil} := 4\cdot\frac{m_{oil}}{\pi\cdot D_{helix}\cdot\mu_{oil}} \qquad Re_{D.oil} = 150.221$$

$$Nu_{D.oil} := 1.86\cdot\left(\frac{Re_{D.oil}\cdot Pr_{oil}}{\frac{10\cdot m}{D_{helix}}}\right)^{.333}\cdot\left(\frac{\mu_{oil}}{\mu_{s.oil}}\right)^{.14}$$

$$h_{oil} := Nu_{D.oil}\cdot\frac{k_{oil}}{D_{helix}} \qquad h_{oil} = 51.777\frac{kg}{s^3K}$$

Calculation of Reynolds number, Nusselt number, and convection coefficient for the water fluid stream (flowing through the annulus without the coils):

$$Re_{D.water} := 4\cdot\frac{m_{water}}{\pi\cdot\left(D_{cyl.o} + D_{cyl.i}\right)\cdot\mu_{water}} \qquad Re_{D.water} = 1965.012$$

$$Nu_{D.water} := .023\cdot Re_{D.water}^{.8}\cdot Pr_{water}^{.4} \qquad Nu_{D.water} = 12.848$$

$$h_{water} := Nu_{D.water}\cdot\frac{k_{water}}{\left(D_{cyl.o} - D_{cyl.i}\right)} \qquad h_{water} = 86.981\frac{kg}{s^3K}$$

Calculation of the log-mean temperature difference with a correction factor for cross flow with one fluid mixed:

$$P := \frac{T_{oil.o} - T_{oil.i}}{T_{water.i} - T_{oil.i}} \qquad R := \frac{T_{water.i} - T_{water.o}}{T_{oil.o} - T_{oil.i}}$$

$$P = 0.283 \qquad R = 0.071$$

$F := .99$

The correction factor is for a single pass, cross-flow heat exchanger with one fluid mixed and the other unmixed. A single pass is assumed because the temperature of the water remains constant. Because of the constant water temperature, the correction factor has no effect.

$$\Delta T_{lm} := \frac{F \cdot \left[\left(T_{oil.i} - T_{water.o}\right) - \left(T_{oil.o} - T_{water.i}\right)\right]}{\ln\left[\frac{\left(T_{oil.i} - T_{water.o}\right)}{\left(T_{oil.o} - T_{water.i}\right)}\right]} \qquad \Delta T_{lm} = 29.99\,C$$

Calculation of the overall length of the inner coil:

$$q := m_{oil} \cdot C_{p.oil} \cdot \left(T_{oil.i} - T_{oil.o}\right) \qquad q = 485.876\,\frac{kg\,m^2 A}{s^2 K}$$

$$U := \frac{1}{\frac{1}{h_{oil}} + \frac{1}{h_{water}}} \qquad U = 32.457\,\frac{kg}{s^3 K}$$

$$L := \frac{q}{U \cdot \pi \cdot D_{helix} \cdot \Delta T_{lm}} \qquad L = 9.931\,m$$

Actual heat exchanger length is 10.2m.

$$\%error := \left(L - L_{actual}\right) \cdot \frac{100}{L_{actual}}$$

$$\%error = -2.642$$

The calculated coil length is 9.931m which is only 2.64% shorter than the actual length. The error can be attributed to small differences in the calculation of convection coefficients.